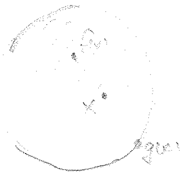
BROUWER'S FIXED POINT THEOREM

THEOREM (BROUWER) Any continuous map $f: D^n \to D^n$ has a fixed point.

PROOF Assume f hasn't got a fixed point, i.e. $f(x) \neq x \ \forall x \in D^n$. Then define a map



$$g: D^n \longrightarrow \partial D^n = S^{n-1}$$

that maps x to the point on ∂D^n by projecting it from $f(x)$. This gives maps

$$S^{n-1} \xhookrightarrow{i} D^n \xrightarrow{g} S^{n-1}$$

such that $g \circ i = id_{S^{n-1}}$ as $g(x) = x$ for $x \in \partial D^n$.
And thus we have maps on cohomology.
$i^* \circ g^* = id_{H^n(S^n)}$. $\lightning$ This isn't possible as $H^n(D^n) = 0$ and thus $i^* \circ g^* = 0$ whereas $id_{H^n(S^n)}$ isn't.

$\square$

REMARK • We can suppose g to be smooth, as we can find a polynomial expression for it, or approximate a ~~smooth g map by~~ continuous ~~ones~~ by smooth ones.